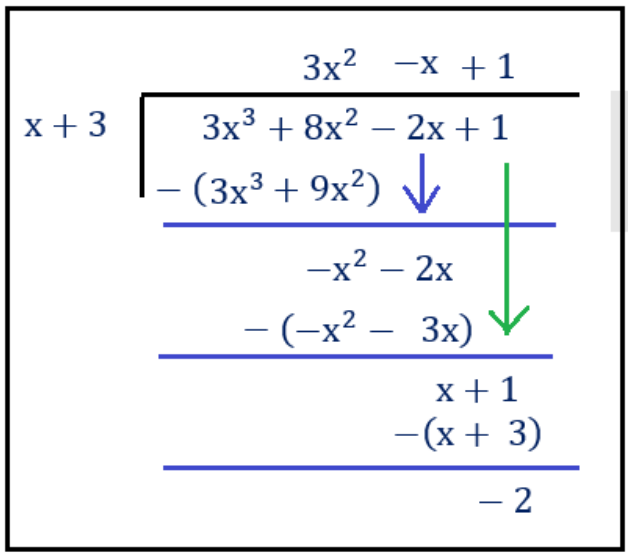
Section 4.3

Section 4.3

(56) Divide by long and synthetic division: $\frac{3x^3 + 8x^2 - 2x + 1}{x+3}$.

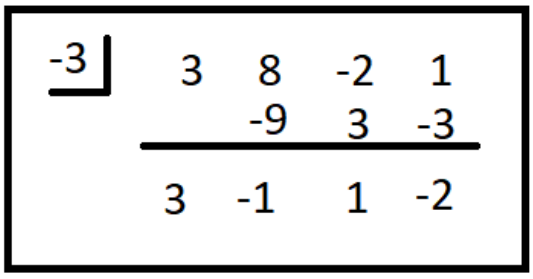
56

Long Division Method:



Answer: $3x^2 - x + 1 - \frac{2}{x+3}$

Synthetic Division Method:



Answer: $3x^2 - x + 1 - \frac{2}{x+3}$

57

(57) Divide: $\frac{x^3 - 2x^2 + 2x - 5}{x^2 + 1}$

$$
\begin{array}{r|l}
 & x - 2 \\
\hline
x^2 + 1 & x^3 - 2x^2 + 2x - 5 \\
 & -(x^3 \quad + x) \\
\hline
 & -2x^2 + x - 5 \\
 & -(-2x^2 \quad - 2) \\
\hline
 & x - 3
\end{array}
$$

Answer: $x - 2 + \frac{x-3}{x^2+1}$

58

(58) Determine the maximum number of (a) real zeros, (b) x-intercepts and (c) relative extrema (turning points) of the function $f(x) = ax^{10} + bx^5 + cx^2 + d$.

Part-a

The degree of the polynomial is 10 so the maximum number of real zeros is **10.**

Part-b

Based on part-a, the maximum number of x-intercepts is also **10**

Part-c

Since the degree is 10, the maximum number of turning points is 10-1 = **9**

59

(59) Form $P(x) = d(x)\,Q(x) + R(x)$ using Exercise 57.

According to Exercise 57,

$P(x) = x^3 - 2x^2 + 2x - 5$

$d(x) = x^2 + 1$

$Q(x) = x - 2$

$R(x) = x - 3$

Therefore,

$$x^3 - 2x^2 + 2x - 5 = (x^2 + 1)(x - 2) + x - 3$$

60

(60) Use the Remainder Theorem to show $f(7) = 188$ if $f(x) = 2x^3 - 11x^2 + 8x - 15$.

We need to show that the remainder is 188 when $f(x) = 2x^3 - 11x^2 + 8x - 15$ is divided by $x - 7$.

Using synthetic division,

7	2	-11	8	-15
		14	21	203
	2	3	29	188

More on next page

Section 4.4

Section 4.4

(61) Use synthetic division to prove $x-5$ is a factor of $f(x) = 2x^3 - 11x^2 + 8x - 15$.

61

5	2	-11	8	-15
		10	-5	15
	2	-1	3	0

$x - 5$ is a factor because the remainder in the calculation above is zero

62

In Exercises 62-63, write the complete factored form of f.

(62) $f(x) = -3x^4 + x^3 + 21x^2 + 11x - 6$, given that $-2, -1, \frac{1}{3}$ and 3 are zeros.

We have

$x = -2,\ x = -1,\ x = \frac{1}{3}, x = 3$

Transpose the constant to the left side for each equation

$x + 2 = 0,\ x + 1 = 0,\ 3x - 1 = 0, x - 3 = 0$

Multiply all the left sides

$(x + 2)(x + 1)(3x - 1)(x - 3) = 0$

Multiply both sides by -1 because the leading coefficient is a negative number

$-(x + 2)(x + 1)(3x - 1)(x - 3) = 0$

Hence, the complete factored form is $-(x + 2)(x + 1)(3x - 1)(x - 3)$

63

(63) $f(x) = 4x^3 - 23x^2 + 26x + 8$, given that 2 is a zero.

To get the other factors, we divide the polynomial by $x - 2$ using synthetic division

2	4	-23	26	8
		8	-30	-8
	4	-15	-4	0

The quotient is $4x^2 - 15x - 4$ which can be factored into $(x - 4)(4x + 1)$

Therefore, the complete factored form is $(x - 2)(x - 4)(4x + 1)$

64

(64) Use the Rational Zero Test and synthetic division to find the complete factored form of $f(x) = x^3 + 10x^2 + 31x + 30$.

We cannot have a positive root because all the signs are positive.
The possible rational roots based on the constant term 30 are -1, -2, -3, -6, -10, -15, and -30.

-2	1	10	31	30
		-2	-16	-30
	1	8	15	0

Since the remainder is zero when the polynomial is divided by x+2, we conclude that x+2 is one of the factors

To get the other factors, we factor the quotient $x^2 + 8x + 15$ into $(x + 3)(x + 5)$

Therefore, the complete factored form is $(x + 2)(x + 3)(x + 5)$

65

(65) State the x-intercepts of f in Exercise 64.

Equate each factor to zero and solve for x.

The x-intercepts are -2, -3, and -5 at coordinates (-2, 0), (-3, 0), and (-5, 0), respectively.